Developing by Key Competences Mathematics Education

Lifelong Learning Programme

Concepts for teaching and learning in class
Deliverable 2.1

This paper outlines the Key Competences in relation to the mathematics education research in class. Specifically, it is attempted to give a definition for each key competence and to accompany it with approaches for further enhancement.

Communication in mother tongue

One of the main concerns of mathematics teaching should be the flexible transition from everyday language to the language used in mathematics:

- Each child has to have the possibility to express his/her own thoughts and feelings through a mathematical approach.
- It is a long journey to reach formal mathematics. This may start from early years when students are asked to add for example 3+2. In such cases students may be asked to tell a story which represents this mathematical sentence. Teachers should try to break the separation that exists between natural language and mathematics.
- When using mathematical language, students need to realize that they need to be precise and accurate. The aim will be to bring precision in their language. This is of great essence and necessary when students will make a transition to more typical and formal mathematics.
- When referring to communication it needs to be stressed that this should be achieved both in written and spoken language.
- Communication in mother tongue should be used in the classroom for students to communicate and understand their teacher as well as their peers.

Approaches

- *Learning journal*: Students may be asked to write a journal about the mathematics they learn in the classroom.
- *Poster or Presentation*: Students should be given the opportunity to present their mathematical approaches, solutions, projects to others in the form of a presentation or a poster. Of course, such activities will also enhance students' social and communication skills as well as their mathematical skills.
- *Mathematisation of everyday life*: Such activities aim to connect mathematical language to mother tongue and everyday life. Through the process of mathematisation students are asked to represent a real-world situation symbolically (in mathematics language) by describing, identifying, formulating and visualising the mathematical problem in their own way; and then moving back by make sense of the mathematical solution in terms of the real solution, including the limitations of the solution.
- *Reformulation activities:* Students are asked to express the same mathematical idea in different ways.
- *Projects*: Students may develop communication in their mother tongue if they are working on their own projects.

Communication in foreign language

One may think that communication in a foreign language may be something that does not have a place in the mathematics classroom. However, this is far from the truth. Foreign language may be a way of describing the communication in another language which students are not familiar with. This key competence is also necessary to be developed when students are asked to communicate with a programming language.

Approaches

- *Learning to do mathematics in foreign languages:* KeyCoMath creates a framework for teachers and teacher-students in Europe to exchange teaching, learning and assessment material. Some "learning/assessment scenarios" are exchanged in the original language, some are exchanged in English as lingua franca. In this process not only "pure mathematics" is transported, but there is also a substantial implicit exchange of cultural background. Thus, any person who works with the material deepens his/her consciousness of the rich European culture.
- *Learning to communicate with the programming language*: Students may be asked to write or read something in programming language. Of course this may be done in various levels of education and in various levels of difficulty.

Digital competence

Digital competence allows students to develop intuitions about various mathematical concepts. Students need to know and also use various mathematical programs. Furthermore, when referring to digital competence we also needs to raise the issue that students also need to learn how various machines work.

Approaches

- *Using any kind of software system*: Asking students to work with any kind of software system increases and improves students' mathematics education.
- *Using modern software systems*: Students must be given the opportunity to work with modern software systems.
- *Virtual reality in mathematics*: Students must also be given the opportunity to work in developing mathematical ideas with the use of virtual reality.

Learning to learn

We set off from the principle that one learns a lot of things in mathematics if they are given the opportunity to solve a lot of problems. Working on problems teaches students the importance of persistence when dealing with a mathematical problem and the importance to develop reasonable arguments. Through problem solving students learn to learn.

Approaches

- *Reflection activities*: Through problems solving students should be asked to reflect on their own learning.
- *Exploration activities*: Mathematics discipline is a good environment for exploring. This gives the opportunity to students to explore and learn by themselves.
- *Plethora of problems*: Students should be given the opportunity to solve different types of problems, for instance, easy and hard problems, problems that can be solved if you are working backwards.
- *Time*: It is important to stress that learning to learn needs time, and time needs to be allocated to this purpose. Students should be given the time to reflect on activities and learn by themselves.

Social Competence

Social competence may be developed through the interaction and communication that students have in the classroom both with their peers and their teacher.

Approaches

- *Listen, Talk, Write, Understand, and Communicate*: Students should be given the opportunity to communicate in verbal and written form. They should learn how to listen carefully to other, to talk with precision and make themselves understood by others.
- *Accept different solutions and respect arguments*: Students must learn to respect the opinions and arguments provided by other people and also accept different solutions.
- *Co-action when communicating*
- *Accuracy in ones communication*: Insist on accuracy in various activities.
- *Dialogic learning*: Teaching should involve the approach of dialogic learning.

Sense of initiative

Sense of initiative may be developed if students realize through the various activities in the mathematics classroom that different approaches may be used. Students may be asked to solve a problem or carry out an assignment where limited or no specific directions are given. Such scenarios, force students to take their own initiative.

Approaches

- *Problems with different solutions*
- *Problems which can be solved with different approaches*
- *Open problems*

Mathematical competence

We left mathematical competence at the end since this encompasses all the previous key competences. If we manage to build an environment where all the previous key competences will be developed this will contribute to the mathematical competence. Mathematical competence is not simply to do calculations but to develop various mathematical concepts, processes and competences as the ones described above.

Quality standards for learning/assessment scenarios

A problem, which is perfectly suited for activating pupils to develop new terms and concepts, can be inapplicable for recapitulation and even more for performance reviews. On the other hand, an 'appropriate' problem in a test may often be unsuitable in an equal format to initialize learning processes. In this case, a separation between problems for studying (= **assignments**), for example discovering and practicing, and problems for assessment (= **test tasks**), with which pupils and teachers can identify performances and appraise competencies, become apparent. Thus, an exercise must always be judged with regard to its function.

A very simple example to illustrate the differentiation between assignments and test tasks:
A test task could be: "Calculate 49*51."
Conversely, as an assignment one could formulate: "Tell me how you calculate 49*51."

When pupils are in a learning process you also may assess their competences there. Assessment is possible just by observing pupils when working or by analyzing their results in the learning process.

The following table characterizes assignments and test tasks. However, it only shows the "ends" of a continuous spectrum. Good teaching includes elements of both types and good assessment also includes elements of both types.

Assignments	**Reality**	**Test Tasks**
aim at openness, divergence, processes, variety of solutions		aim at assessability, convergence, visible results
should consider errors as a chance for learning		rather require the prevention of errors
allow or rather support cooperation and communication		focus on individual performance
are exercises, for example to • explore, detect, invent* • collect, protect, systematise • practise, network, recapulate		are exercises, for example to • assess performance** • apply (experience of competence) • self-review
Important is, what's going on in the brain.		Important is, what pupils show.

***Explore, detect, invent:**

- The exercise sees the **pupil as a free actor**. Motivation results from the experience of **autonomy**, **social embeddedness** and **competence** (Deci, Ryan).
- The exercise has to be **easily accessible**, i.e. it relies on existing knowledge and experience or it is integrated into demonstrative situations.
- The exercise poses **challenging questions** (e.g. via paradoxes or contradictions).
- The exercise mostly consists out of an **open situation**, in which (research) questions still have to be developed.
- The exercise allows **different ways of approaches** and problem-solving strategies and/or **several results**.

- The exercise allows **working mathematically on different levels**, so that each pupil can develop mathematical competences on his/her individual level.
- The exercise requires that, first of all appropriate **solution strategies are developed** and devised, which lead to a result.
- The exercise allows **variation** and, if necessary, **simplification** of the problem.
- The exercise is scientifically **meaningful** and brings forth concretization of concepts and essential scientific ideas.

****Performance assessment**:

- The exercise **focuses** on the knowledge and ability that is to be assessed and does not add further secondary aspects.
- The exercise is phrased for the pupils to understand what is expected from them. The language should be simple and the tasks **clearly phrased**.
- The exercise should allow to assess **either skills or knowledge**.

A good or rather meaningful "**learning/assessment environment"**

- is formulated comprehensibly,
- fits to the current learners' state of knowledge,
- demands what was practiced before,
- forms a complex learning arrangement that actively links various competencies; integrates productive and receptive skills,
- encourages students in a motivating way to deal actively and problem-oriented with/in a situation,
- considers the actual use and level of language,
- gives priority to working with the content of the exercise; formal and structural aspects are of secondary importance,
- acts cognitively activating,
- includes acquiring knowledge of strategies,
- allows self-assessment,
- encourages the pupils to reflect on their increase of knowledge,
- encourages the pupils to reach results in reasonable social forms,
- offers the possibility for internal differentiation,
- leads to a concrete result/product, with which the achievement becomes visible.

Measuring key competences:

Excercises for measuring students key competences should

- be reduced to the **important aspects** (no complex excercises),
- be **open**, in order for individual solution processes to be possible and no particular solution process becomes apparant
- encourage and request **pupils' productions** (drawings, reasons)
- demand **reflections** such as descriptions, explanations or reasons.

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