

Random Experiments

Investigating Probability

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Intention

In this learning environment, different random experiments will be tested with dice and coins to enable the pupils to become familiar with the term probability in both single-step experiments and compound experiments. The pupils should recognize the connection between statistical probability and traditional probability and be familiar with the expected value and the effects of random fluctuations. About six lessons are recommended for this learning environment.

Background of Subject Matter

In the lesson, the following topics will be covered:

- Statistical probability
- Presentation and distribution of values in random experiments
- Chances and probability of success
- Computer based simulation of random experiments
- Random fluctuation

Methodical Advice

The pupils will work together in groups of two to three in order to solve the tasks. The solutions will then be presented and discussed in class.

The Excel documents can be downloaded at: www.KeyCoMath.eu

The pupils can otherwise develop these themselves in cooperation with the teacher for computer science. As a result, more lessons will be required to complete the tasks.

Practical Experience from Lessons

Is the Die Marked?

The pupils noticed that it is possible to throw three sixes in a row. Some argue with numerical probability, others with their own experience or special throwing techniques.

① Ich finde, dass nicht geschummelt wurde.
 Es ist durchaus möglich, dass 3x in Folge eine 6 gewürfelt werden kann, wie es mir auch einmal passiert ist.
 Es ist zwar unwahrscheinlich und sehr selten, aber möglich. Die Chance ist gleich hoch wie 3x eine 1 zu werfen! Weil die 1 1x vorkommt und auch die 6 nur 1x.
 Glückssache!

1. Wenn Franz dreimal 6 Gewürfelte hat hat er Glück gehabt
 Bei 3 mal 2 Würfeln werden gleiche Zahl ist 18: 216
 Er hat sehr viel Glück gehabt.
 $100\% : 216 = 0,462\%$
 $\frac{100}{216} = 0,462\%$

Two Time Throw

Flipping a Coin: Most of the pupils are able to name the four possible outcomes. However not all know that all four events are equal in probability.

Throwing the Dice: The argumentations about throwing the dice twice are quite different. Most of the pupils acknowledge that the probability of a six in one throw is $\frac{1}{6}$. Some can even translate this to two throws.

Many pupils have problems with giving valid estimations. The pupils discuss their estimations about the 600 throws with two dice in groups of four to five. Some groups argue with their experience from the previous experiments, other groups argue with probability in general.

② Unser Versuch Möglichkeiten:
 a) 1. x = Zahl 1. Zahl | 1. Kopf | 1. Zahl | 1. Kopf
 2. x = Kopf 2. Zahl | 2. Kopf | 2. Kopf | 2. Zahl
 b) Wahrscheinlicher ist nichts. Es haben immer beide (Kv. 2) 50% die Möglichkeit. Gleich wahrscheinlich.
 c) Bei jedem Würfeln gibt es $\frac{1}{6}$ mal die Möglichkeit, dass eine 6 vorkommt. $\frac{1}{6}, \frac{1}{6}, \frac{1}{6}$
 d) Meiner Meinung nach 25%. 2 mal. also $\frac{2}{6}$. 2x 3x

a. 2 x Kopf
 2 x Zahl
 1 x Kopf 1 x Zahl
 b. Das kann man nicht sagen, aber wahrscheinlich ist es verschieden, weil wir es versucht haben und auf 2x war es 3 verschieden.
 c. Es wird möglich, dass man 2 sechser würfelt, 1 oder keinen

Throwing Two Dice

The question about a fair game is answered very differently.

a) Für mich ist ein „faires Spiel“, wenn die Chance zu gewinnen nicht zu klein ist und wenn die Gewinnsumme angemessen ist.

a) fair sein ist eh problematisch. Wenn man Fair spielen liiert macht keine Verlust und keine Gewinn also z.B. spielt man 2 spielen

Also wichtig
 gewonnen
 also
 10€
 Zeit
 Verlust

The question whether this game was fair was answered mostly empirically.

Anzahl der Würfelseiten	Gewinn (2x6)		Verlust (1x6 - 0x6; 0x6 - 0x6)	
	1. Spiel	2. Spiel	1. Spiel	2. Spiel
1				
2			X	X
3			X	X
4			X	X
5			X	X
6			X	X
7			X	X
8			X	X
9			X	X
10			X	X
11			X	X
12			X	X
13			X	X
14			X	X
15			X	X
16			X	X
17			X	X
18			X	X
19			X	X
20		X	X	X

1. Spiel: Wir hatten 20€ Verlust!
 2. Spiel: Wir hatten 10€ Verlust! 20 x 1€ gegeben
 10€ x 1 gewonnen
 Man müsste bei 20 Spielen 2 x Doppel 6 Würfeln um keinen Verlust zu machen. Das ist sehr unwahrscheinlich!

Das Spiel ist unfair, denn die Wahrscheinlichkeit, dass man 2 mal eine 6 würfelt ist ziemlich klein. Bei 36 Spielen ist die Wahrscheinlichkeit, dass man eine doppelte 6 würfelt gleich 1. Man macht Verlust von insg. 26€, da man ein mal 10€ gewinnt. Da man bei jedem Wurf 1€ zahlen muss und nur bei einer doppelten 6, 10€ gewinnt ist das Spiel sehr unfair.

ANNA		DENIS		ROMINA	
Verlust	Gewinn	Verlust	Gewinn	Verlust	Gewinn
###		###	###	###	

Developing Game Variants

The variants were very different both in completeness and complexity.

Spiel :

Sack \rightarrow 5 Kugeln ① ② ③ ④ ⑤

~~Wenn der Spieler~~

Der Spieler zieht 2 Mal 2 verschiedene Kugeln (ohne Zurücklegen), wenn die Summe der Zahlen auf den gezogenen Kugeln eine Zahl im Sack ergibt so hat der Spieler gewonnen. Andernfalls hat er verloren.

Das heißt: Der Spieler gewinnt, wenn er folgende Zahlen zieht:

$$\begin{aligned} ① + ② &= ③ \\ ① + ③ &= ④ \\ ① + ④ &= ⑤ \\ ② + ③ &= ⑤ \end{aligned}$$

$P = \frac{\text{Anzahl der günstigen Möglichkeiten}}{\text{Anzahl aller Möglichkeiten}} = \frac{4}{10} = \frac{2}{5}$

Performance Rating

The developed material will be handed in together with the work and will be used in rating the performance.

Random Experiments

1 Is the Die Marked?

While playing “Mensch ärgere Dich nicht“, Franz has thrown three sixes in a row. Peter is annoyed and claims that Franz has used a marked die. What do you think about this? Argue.

2 Two Time Throw

Flipping a Coin

A coin is flipped twice. It is being noted down which side is up. Try to describe all possible outcomes. Which outcome do you think is more probable: Twice the same side or two different sides?

Throwing the Dice

A die is thrown twice. How would you estimate the chances of throwing a certain number of sixes? Argue.

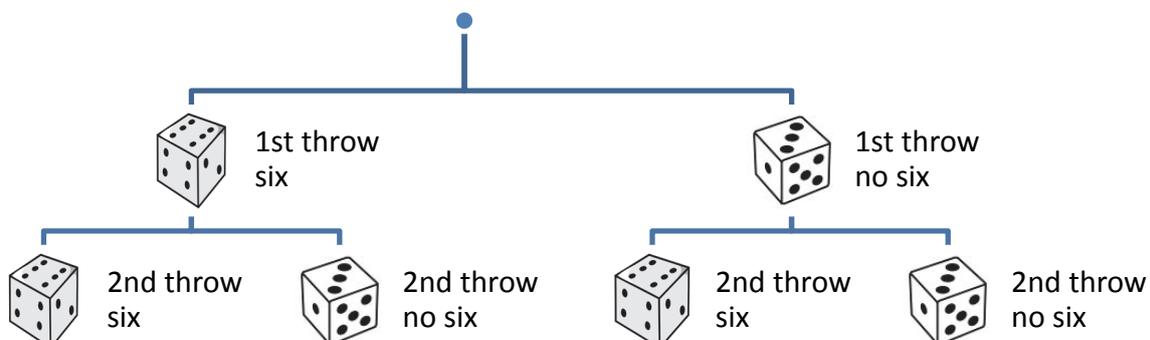
Run the experiment with the two dice 50 times and report your outcome to the teacher. Has your opinion changed or was it confirmed?

The “Throwing the Dice“-Experiment is being run 600 times. Estimate, how often the following events occur:

- A: Six in the first throw.
- B: No six in the first throw.
- C: Six in the second throw.
- D: Six in both throws.
- E: At least one six is thrown.

Think of at least two more events and estimate their occurrence.

Copy the tree diagram below into your exercise book and label the different events with their empirical probability.



Throwing a Lego Brick

A square Lego brick is thrown once. Determine the probability of the following events:

- A: The brick falls onto the base.
- B: The brick falls onto a side.
- C: The brick falls onto the side with the nubs.

Argue your claims.

3 Sum of the Dice – Simulation with Excel

Two dice are thrown.

Which sums of the dice (sums of the number of points) are possible and which sums do you think are very likely and which less likely? Argue your claim.

Give the probabilities of the different sums of dice.

It is possible to simulate this experiment with Excel. Open the file „Random_Experiments.xlsx“. Using „F9“ you can throw two dice 500 times.
(Possibly, iterations have to be activated in the Excel options for formulae.)

Run this experiment 30 times in order to have 15000 throws. Compare the result with your estimation. What do you notice?

4 Throwing Two Dice

You play against a bank:

- Bet: 1 €
- A die is thrown twice. If two sixes are thrown, the player wins 10 €.

Think about if this is a fair game. At first, discuss with your neighbour what a “fair game“ might be. Play the game 20 times with your neighbour. Note down the throws and the prizes.

Collect all outcomes in class and calculate the average prize.

5 Developing Game Variants

Vary the game in task 4.

Possible variations:

- Bets and prize money
- Conditions for winning
- Flipping a coin instead of throwing the dice
- Number of throws

Hand in two of your developed games with an analysis of their profitability.