

# Important Lines in Triangles

## A Computer-Assisted Lesson

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### Intention

The pupils construct the most important features of triangles with the help of GeoGebra and study their characteristics by changing the shape of the triangles. Four to six lessons are recommended for this tasks depending on the pupils' knowledge of GeoGebra.

### Background of Subject Matter

The most important points in triangles (Point of intersection of perpendicular bisectors as circumcentre, point of intersection of angle bisectors as incentre, centroid (centre of mass), and point of intersection of altitudes as orthocentre) are being constructed. The location of these points varies depending on the shape of the triangle; three out of the four are located on the Euler line. GeoGebra enables the pupils to test various cases and through this draw conclusions. Additionally, faster pupils can try to find the Fermat point.

### Methodical Advice

In order to work with this learning environment, pupils require knowledge of the following terms:

- Perpendicular bisector
- Angle bisector
- Median
- Altitude
- Distance

The first task can be used for demonstration purposes in order to explain GeoGebra's tools. Pupils ought to work alone or together with a partner.

### Performance Rating

The pupils solve the tasks and write down their thoughts and results on the computer. The produced graphs from GeoGebra will be added to the document. This document will then be graded according to content, clarity, and the proficiency in construction and argumentation.

# The Most Important Lines in Triangles

For each task, create an own window in GeoGebra which you can access at any time.

## 1 Altitudes

Construct a random triangle and draw in the three altitudes. Construct their point of intersection  $O$ . Move the vertices of the triangle so that the orthocentre  $O$  is relocated as a result. Note down how the location of the orthocentre moves in relation to the shape of the triangle.

## 2 Perpendicular Bisectors

Construct the perpendicular bisectors of a triangle. Label their point of intersection as  $C$ . Move the vertices of the triangle so that the point of intersection  $C$  is relocated as a result. Note down how the location of  $C$  changes in relation to the shape of the triangle.

## 3 Angle Bisectors

Construct the angle bisectors of a triangle. Label their point of intersection as  $I$ . Is it possible that  $I$  is located outside of the triangle? Argue.

## 4 Medians

Construct the line segments from the midpoints of the sides to the opposite vertices. Label their point of intersection as  $S$  and study its location.

## 5 Circles and Relations

Which one of the points has the same distance from all three vertices of the triangle?

Which one of the four points has the same distance from all three sides of the triangle?

Measure the distance between the vertices and the four points of intersection and the distance between the four points of intersection and the three sides of the triangle. What do you notice?

Additional Task: Is there a point for which the sum of the distances to the vertices is minimal?

## 6 Connection between the Different Points of Intersection

Construct a triangle that covers most of the screen. Draw the now familiar special lines and their points of intersection into the triangle. Fade out the special lines so that a clear picture remains. Move the triangle and track the motion of the four points of intersection. What do you notice?

Are you able to move the vertices in the manner that all four points of intersection are situated on a line?