

# Functions and their Derivatives

## Courses of Functions, Monotony, and Curvature

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### Intention

The Aim of this worksheet is to make pupils familiar with the connection between a function and its derivatives. No calculations are needed in order to solve the tasks. Practical experience shows that about 50min are needed to complete the worksheet.

### Background of Subject Matter

Multiple tasks are aimed at an understanding of the relation between monotony and curvature.

### Methodical Advice

The pupils should solve the tasks alone or together with a partner. The tasks' level of difficulty is average.

### Performance Rating

In an exam, it is easily possible to test if the students have understood the relations and are able to argue their claims through varying the tasks.

# Functions and their Derivatives

May function  $f$  be continuous and differentiable.

## Task 1

May function  $f$  be strictly monotonically decreasing in the interval  $I$ . What happens to functional value  $f(x)$  in the interval if the  $x$ -value of the function is decreasing? Argue.

## Task 2

In a right curve, may the function  $f$  have the slope 4 in  $x_0$ . Which slope will  $f$  have in  $x_1$  in this right curve, if  $x_1$  is located to the right of  $x_0$ ? Argue.

## Task 3

May  $f''$  possess the value -5 in  $x_0$ . What does this tell you about  $f'$  and  $f'(x_0)$  in a neighbourhood of  $x_0$ ? What can be said about  $f(x_0)$ ? Argue.

## Task 4

May  $f'$  intersect the  $x$ -axis in  $x_0$  from above. Which special point is  $x_0$ ? Argue.

## Task 5

If necessary, correct the following argumentation:

In a right curve of a function  $f$  the slope of the tangent decreases if  $x$  moves from left to right. As the derivate of a function describes the slope of the tangent in a particular point,  $f'$  and  $f''$  are strictly monotonically decreasing in the right curve. Argue if you have changed the argumentation.

## Task 6

If possible, draw a sketch of a section of a function  $f$  which has a left curve and is strictly monotonically decreasing. What can be said about  $f''$  in this part of the curve? Argue.

## Task 7

May function  $f$  change from a right curve into a left curve in  $x_0$ . Additionally, may  $f$  be strictly monotonically increasing close to  $x_0$ . Is this possible? Argue with the help of a sketch.

## Task 8

May function  $f$  change from a right curve into a left curve in  $x_0$ . Additionally, may  $f$  be strictly monotonically decreasing close to  $x_0$ . Is this possible? Argue with the help of a sketch.

### Task 9

May  $f''$  be monotonically decreasing in an interval and possess an x-intercept in this interval. What can you conclude about  $f$  in this interval? Argue.

### Task 10

May  $f'$  be strictly monotonically increasing in an interval  $I$  but may  $f'$  possess only negative functional values in this interval. Sketch the course of  $f$  in the interval and argue your solution.