

Functions and their Derivatives

Courses of Functions, Monotony, and Curvature

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Intention

The Aim of this worksheet is to make pupils familiar with the connection between a function and its derivatives. No calculations are needed in order to solve the tasks. Practical experience shows that about 50min are needed to complete the worksheet.

Background of Subject Matter

Multiple tasks are aimed at an understanding of the relation between monotony and curvature.

Methodical Advice

The pupils should solve the tasks alone or together with a partner. The tasks' level of difficulty is average.

Performance Rating

In an exam, it is easily possible to test if the students have understood the relations and are able to argue their claims through varying the tasks.

Functions and their Derivatives

May function f be continuous and differentiable.

Task 1

May function f be strictly monotonically decreasing in the interval I . What happens to functional value $f(x)$ in the interval if the x -value of the function is decreasing? Argue.

Task 2

In a right curve, may the function f have the slope 4 in x_0 . Which slope will f have in x_1 in this right curve, if x_1 is located to the right of x_0 ? Argue.

Task 3

May f'' possess the value -5 in x_0 . What does this tell you about f' and $f'(x_0)$ in a neighbourhood of x_0 ? What can be said about $f(x_0)$? Argue.

Task 4

May f' intersect the x -axis in x_0 from above. Which special point is x_0 ? Argue.

Task 5

If necessary, correct the following argumentation:

In a right curve of a function f the slope of the tangent decreases if x moves from left to right. As the derivate of a function describes the slope of the tangent in a particular point, f' and f'' are strictly monotonically decreasing in the right curve. Argue if you have changed the argumentation.

Task 6

If possible, draw a sketch of a section of a function f which has a left curve and is strictly monotonically decreasing. What can be said about f'' in this part of the curve? Argue.

Task 7

May function f change from a right curve into a left curve in x_0 . Additionally, may f be strictly monotonically increasing close to x_0 . Is this possible? Argue with the help of a sketch.

Task 8

May function f change from a right curve into a left curve in x_0 . Additionally, may f be strictly monotonically decreasing close to x_0 . Is this possible? Argue with the help of a sketch.

Task 9

May f'' be monotonically decreasing in an interval and possess an x-intercept in this interval. What can you conclude about f in this interval? Argue.

Task 10

May f' be strictly monotonically increasing in an interval I but may f' possess only negative functional values in this interval. Sketch the course of f in the interval and argue your solution.