Arguing and Proving Assumptions:
Teaching Concept for Circular Sectors

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Intention and Aims

Over the course of the lesson (90 min), the pupils should become aquainted with the definition of the term circular sector. In cooperation between teacher and pupils, assumptions about the mathematic formulas for the area and arc length of a circular sector ought to be made. Focus of the tasks should be the reasoning and proving of the elaborated theories.
Finally, the pupils should work with the newly acquired formulas.

Background Knowlegde

In order to derive the formulas for a circular sector, pupils require knowledge of the equivalent formulas for area and circumference of a circle.

Methodical Advice

In the beginning of the lesson, explicit examples (e.g. slices of cake) should be given in order to aquaint the pupils with the term circular sector.

The remaining part of the first half of the lesson should be organized in the following manner: the pupils should be given a task which guides them through the process of deriving the formula for the area of a circular sector from the formula for the area of a circle. Through this, the pupils are given the opportunity to discover the connection between central angle and area on their own. The pupils' solutions should then be discussed and argued in class.

The second half of the lesson should be structured equivalently in order to help the pupils with deriving the formula for the arc length from the formula for the circumference of a circle. The remaining time should be used for working with the new formulas on the worksheet provided below. Most of these tasks are again focusing on the competence of argumentation.

As a homework, the pupils should continue the tasks on the worksheet.

Area and Arc Length of a Circular Sector

1. A circular sector has area A, arc length b, radius r and circular angle α. Calculate the missing quantities.
   a) $A = 170cm^2, \ b = 20cm$,
   b) $r = 11,8cm, \ \alpha = 60°$,
   c) $r = 1cm, \ b = 1cm$.

2. Right or wrong? Argue. (The other quantities remain unchanged)
   In a circular sector
   a) twice the circular angle corresponds to twice the arc length,
   b) half the radius leads to half the arc length,
   c) three times the radius corresponds to three times the arc length,
   d) four times the area equals twice the radius,
   e) twice the arc length corresponds to four times the area if the radius is not changed.

3. A pizza chef bakes pizzas with two different diameters. The large pizza's diameter is twice the diameter of the small pizza. The chef cuts his pizzas into 8 equal slices. He sells a slice of the small pizza for 2€. For how much money does he sell a slice of the large Pizza? Does this make sense? Argue.

4. A gazell is running through the savanna at a speed of 6 kilometers per hour. Which area of a circle could it circumnavigate in eight hours if it kept up the pace?

5.
   a) From a 5cm high chocolate cake with a diameter of 26cm, a slice with the circular angle 40° is cut out. How much energy is consumed with this slice, considering that $1cm^3$ of this cake weighs 0,5g and 100g of the chocolate cake contain 1500 kilojoules? Compare your result with the average daily requirement of an adult.
   b) Another cake of the same type is 25% higher. By how many percent is the energy of a slice (circular angle 40°) increased? Argue.
   c) A third cake of the same type has a diameter which is 25% larger than the first cake but has the same height. By how many percent is the energy of a slice (circular angle 40°) increased? Argue.
   d) Compare your result from subtask c) to task 2d).